Section 1.1

Functions

A real-valued function f of a real-valued variable x assigns to each real number x in a specified set of numbers, called the **domain** of f, a unique real number $f(x)$, read "f of x". The variable x is called the **independent variable**, and f is called the **dependent** variable. A function is usually specified **numerically** using a table of values, **graphically** using a graph, or **algebraically** using a formula.

Domain

The set of all values of the independent variable for which a function is defined is called its **domain**.

Graph of a Function

The graph of a function f is the set of all points $\big(x, f(x)\big)$ in the plane with x in the domain of f.

Piecewise-Defined Function

A function specified by two or more different formulas.

Vertical Line Test

For a graph to be a function, every vertical line must intersect the graph in *at most* one point.

Common Types of Algebraic Functions

Linear	$f(x) = mx + b$	m, b constant
Quadratic	$f(x) = ax^2 + bx + c$	a, b, c constant $(a \neq 0)$
Cubic	$f(x) = ax^3 + bx^2 + cx + d$	a, b, c, d constant $(a \neq 0)$
Polynomial	$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$	$a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ constant
Exponential	$f(x) = ab^x$	a, b constant (b positive)
Rational	$f(x) = \dfrac{P(x)}{Q(x)}$	$P(x)$ and $Q(x)$ polynomials

Problem 1. Evaluate or estimate each expression based on the following table.

x	-3	-2	-1	0	1	2	3
$f(x)$	1	2	4	2	1	0.5	0.25

a) $f(0)$

b) $f(2)$

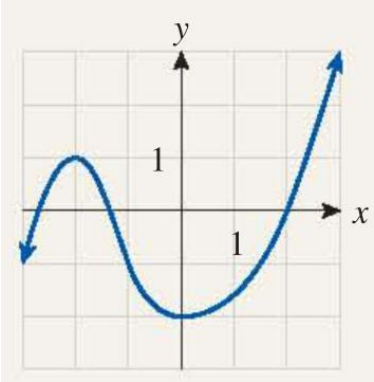
c) $f(1) - f(-1)$

d) $f(1)f(-2)$

Problem 2. Sketch the graph, and find the domain of the following functions:

$$y = x, \quad y = -x, \quad y = x^2, \quad y = x^3, \quad y = \frac{1}{x}, \quad y = \sqrt{x}, \quad y = |x|, \quad y = 5$$

Problem 3. Use the graph of the function to find approximations of the given values.



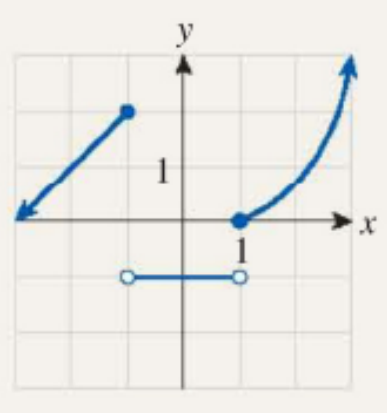
a) $f(-2)$

b) $f(0)$

c) $f(2)$

d) $f(2) - f(-2)$

Problem 4. Use the graph of the function to find the approximations of the given values.



a) $f(-1)$

b) $f(0)$

c) $f(1)$

d) $\frac{f(3)-f(1)}{3-1}$

Problem 5. Sketch the graph of the given function, and evaluate the given expressions.

a) $y = \begin{cases} -x^2, & \text{if } -2 < x \leq 0 \\ \sqrt{x}, & \text{if } 0 < x < 4 \end{cases}$

$f(-1), f(0), f(1)$

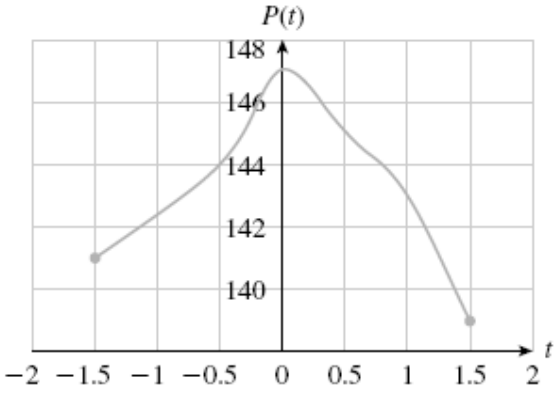
b) $f(x) = \begin{cases} x - 1 & \text{if } x \leq -1 \\ 2x & \text{if } -1 < x < 1 \\ x^3 & \text{if } x \geq 1 \end{cases}$

$f(-1), f(0), f(1)$

Problem 6. The following graph shows an index $P(t)$ of productivity in the US, where t is the time in years and $t = 0$ represents January 2000.

a) What is the domain of P?

b) Estimate $P(-0.5), P(0)$ and $P(1.5)$. Interpret your answers.



Problem 7. Given $f(x) = -3x + 4$, find:

a) $f(-1)$

b) $f(2)$

c) $f(a + b)$

Problem 8. Given $g(x) = 2x^2 - x + 1$, find:

a) $g(-3)$

b) $g(x + h)$

Problem 9. Find and simplify $\frac{f(x+h)-f(x)}{h}$ for the following functions:

a) $f(x) = 2x + 3$

b) $f(x) = x^2 + x$

Problem 10. The following table lists the net sales (after tax revenue) at the Finnish cell phone company Nokia for each year in the period 1995-2001 ($t = 5$ represents 1995)

Year t	5	6	7	8	9	10	11
Nokia Net Sales $P(t)$ (billions of dollars)	8	8	10	16	20	27	28

Find $P(5)$ and $P(10)$. Interpret your answers. What is the domain of P?

Homework for this section: Read section 1.1. Watch any videos (marked with  in the e-book)

Also, do the tutorials (marked with  in the e-book). Come to class with at least two questions related to what you read/watched. Do the following problems in preparation for the quiz: #1-15 (odd), 25, 35, 39, 43, 49